Towards a Theory of Part

My aim in this paper is to outline a general framework for dealing with questions of part-whole. My approach is very different from the more conventional approaches to the subject. For instead of dealing with the single notion of mereological part or sum, I have attempted to provide a comprehensive and unified account of the different ways in which one object can be a part of another. Thus mereology, as it is usually conceived, will become a relatively small aspect of a much larger subject.¹

My discussion has been intentionally restricted in a number of ways. In the first place, my principal concern has been with the notion of absolute rather than relative part. We may talk of one object being a part of another relative to a time or possible world (as when we say that the tire was once a part of the car or that the execution of Marie Antoinette was a part of the French Revolution) or in a way that is not relative to time or possible world (as when we say that this pint of milk is a part of the quart or that the letter ‘c’ is part of the word ‘cat’). Many philosophers have supposed that the two notions are broadly analogous and that what goes for one will tend to go for the other.² I believe this view to be mistaken and a source of endless error. But it is not my aim to discuss either the notion of relative part or its connection with the absolute notion.³

In the second place, I have focused on the ‘pure’ theory of part-whole rather than its application to our actual ontology. Once given a theory of part-whole, there arises the question of how it applies to the objects with which we are already familiar; and although a large element of the interest and justification of the theory derives from its applications, my concern has been more with the abstract development of the theory itself.

Finally, I have only provided the merest sketch of the framework (on which I hope say more elsewhere). Many points are not developed and some not even stated. I have, in particular, said relatively little about the technical foundations of the subject, which are mathematically

¹The material outlined in this paper has been developed over a period of close to thirty years. I have presented it in numerous lectures and seminars, including a seminar at Princeton in 2000, which was attended, among others, by Cian Dorr, Michael Fara, Gail Harman, Mark Johnston and David Lewis; and I am grateful for the comments I received on this and other occasions.

There are some broad and striking similarities between my own views and those conveyed by Mark Johnston [2006] in his paper ‘Hylomorphism’. These similarities include: the adoption of an essentialist framework; the endorsement of what I call ‘naive metaphysics’ (in which foundational considerations are set aside); a pluralist view of part-whole and its application to the abstract sphere, in particular; the idea of artificial or fiat parts, where these are not bound up with the analysis of the whole; the appeal to generative functions as the means by which wholes may be formed from their parts or parts from wholes; the understanding of hylomorphism as a species of essentialism; and the conception of our ordinary ontology as the salient restriction of the theoretically sanctioned ontology. I hasten to add that there are many points in his paper which I would not wish to endorse and, in particular, I do not share his peculiar understanding of a ‘principle of unity’ or its subsequent application to the ‘culling’ of the ontology. It was his paper that led me to write up some of these ideas for fear that they might not be recognized as my own or, worse still, that it might be recognized as my own but only in the mangled form in which they had been reproduced by someone else.

²As in Sider [2001], for example

³The matter is briefly discussed in Fine [1999]
quite distinctive, or about some of the broader philosophical issues to which they give rise. I have given a rough map of the terrain rather than a guided tour, but I hope I have done enough to bring out the interest of the approach and to make clear how a more systematic and philosophically informed account might proceed.

The paper is in three main parts. The first (§§1-4) sets up the general framework which is employed in the rest of the paper and attempts to defend the two central theses of pluralism and operationalism upon which the framework depends. The second (§§5-8) applies the framework to the classical notion of summation and to a wide range of other mereological notions that follow in its wake. The third part (§§9-10) discusses the connection between the notions of part and priority. It is my belief that a whole may sometimes be prior to its parts and that it is only through understanding how this is so that we can hope to provide a fully adequate theory of part-whole.

§1 The Intuitive Notion of Part

One object may be a part of another - some thunder a part of the storm, for example, or the shell a part of a nut. When one object is a part of another, there is a sense in which it is in the other - not in the sense of being enclosed by the other, as when a marble is in an urn, but more in the sense of being integral to the other. When parts are in question, it is also appropriate to talk of a given object being composed of or built up from the objects that it contains. Thus a storm may be composed of various occurrences of lightning and thunder, while an urn is not composed - even in part - of the marbles that it contains.

We may perhaps make the sense of containment characteristic of part especially vivid by considering what happens to an object when a part of it is replaced. For, as a general rule, the object within which the replacement has been made will change - either in the radical sense of being different from what it was or in the less radical sense of being different from how it was. And conversely, it is only parts (or objects containing parts) whose replacement can result in change. Thus if my kidney, which is part of me, is replaced with another kidney, then I will have changed with respect to how I was, while if the hat I am now wearing, which is not a part of me, is replaced with another hat, then I will not thereby have changed with respect either to what or to how I was.

§2 Pluralism

According to the pluralist about part-whole, there are different ways in which one object can be a part of another. Thus he may well think that the way in which a pint of milk is part of a quart is different from the way in which the letter ‘e’ is part of the word ‘cat’ or different from the way in which a member is part of a set.

There is a way in which pluralism may be trivially false. For one may simply stipulate that ‘part’ is to have such and such a sense - the sense which it is assumed to have in standard mereology, for example; and so the truth of monism will be guaranteed as long as there is only one way, in this sense, for an object to be a part of another. But my interest is in the intuitive notion of part, not in some stipulated sense, and, although this notion may be subject to further clarification, there remains a genuine question as to whether any reasonable clarification of it will admit of different ways for one object to be a part of another.

There is also a way in which pluralism may be trivially true. For one way to be a part is to be a small part and another way is to be a large part; and everyone can agree that there are small parts and large parts. But being a small part or a large part are what one might call
derivative senses of part, they are to be understood in terms of more basic senses of part; and in considering the question of pluralism, the derivative senses of part should be set aside.

Let us say that a way of being a part is basic if it is not definable in terms of other ways of being a part. A basic way of being a part may not be basic in an absolute sense, since it may be possible to define it in terms of other mereological notions and, indeed, later I shall suggest that this is so. But there is no definition of it in terms of other ways of being a part. Our question, then, is whether there are different basic ways in which one object may, intuitively, be a part of another.

Now on the face of it, there would appear to be a wide variety of basic ways in which one object can be a part of another. The letter ‘n’ would appear to be a part of the expression ‘no’, for example, and a pint of milk a part of a quart; and if these two relations of part are not themselves basic (perhaps through being restricted to expressions or quantities), there would appear to be basic relations of part that hold between ‘n’ and ‘no’ or the pint and the quart. It is also plausible that the way in which ‘n’ is a part of ‘no’ is different from the way in which the pint is a part of the quart. For if the two ways were the same, then how could it be that two pints were only capable of composing a single quart, while the two letters ‘n’ and ‘o’ were capable of comprising two expressions, ‘no’ and ‘on’? For the same reason, the way in which the letter ‘n’ is a part of the expression ‘no’ would appear to be different from the way in which it is a part of the set of letters {‘n’, ‘o’}; and the way in which ‘n’ is a part of {‘n’, ‘o’} would appear to be different from the way in which a pint is a part of a quart, since if four quarts compose a gallon the pints which compose the quarts will compose the gallon in the same way in which they compose the quarts whereas, if four sets compose a further set the members of the sets will not compose the further set in the same way in which they compose the component sets. Thus we would now appear to have three different basic ways in which one object can be a part of another (pint/gallon, letter/word, and member/set); and once these cases have been granted, it is plausible that there will be many more.

Although pluralism would appear to be the more plausible view, it is not the view that has been most widely held. The majority of philosophers currently working in metaphysics have been monists. They have supposed that there is but one (basic) way for a given object to be a part of another; and they have thought that this one way is the relation of part-whole explored in classical mereology, according to which a whole is a mere sum, or ‘aggregate’ or ‘fusion’, formed from its parts without regard for how they might fit together or be structured within a more comprehensive whole.

Of the many putative counter-examples to the monist position, philosophers have only paid serious attention to the case of set-membership; and so let us focus on this one case as being perhaps typical of the rest. There are three successively stronger claims that should be established if the case of sets is to pose a threat to monism: (i) a member of a set is a part of a set; (ii) it is a part of the set in a basic way; and (iii) the basic way in which it is a part of the set is different from the way in which something is a mere part of the sum. (ii) is eminently reasonable given (i); and (iii) may be taken to hold for the reasons already given. So the success of the counter-example turns on (i). Is a member of a set a part of the set?

As I have indicated, there is a strong prima facie case in favor of members being parts. For we do indeed talk of a set containing its members and of its being composed or being built up from its members.

Two main arguments have been given on the other side. According to the first, the relation of part to whole is transitive but, since the relation of member to set is not transitive, the
members of a set cannot be its parts. But this is to confuse two claims - that the relation of member to set is a relation of part and that the members of a set are parts. The first may indeed be denied on the grounds that the relation of member to set is not transitive but that is still compatible with each case of membership being a case of part. Indeed, it may well be thought that the way in which a member is a part of a set is given, not by the membership relation itself, but by the ancestral of the membership relation, where this is the relation that holds between x and y when x is a member of y or a member of a member of y or a member of a member of a member of y, and so. The way in which a member is a part of a set will then indeed be transitive and the relation of member to set will merely correspond to the special case in which the object is directly a part of the whole. It might be thought odd that the relation of part-whole should be disjunctive in this way but, as we shall see, this is the usual case.

The other argument is that it is only in a metaphorical or non-literal sense that we may talk of a set containing or being built up from its members. There is perhaps some kind of analogy between the members of a set and the bricks that make up a wall but that should not confuse us into thinking that we have a relationship of part in both cases.

I would not wish to deny that there may be metaphorical or non-literal uses of part-whole talk. Thus it is sometimes said that the conclusion of a valid argument is contained in the premisses or that the mother or father is ‘in’ the child. But, of course, the conclusion is not literally a part of the premisses and nor is the mother or father literally a part of the child; and not only is this intuitively evident, it is also revealed by the fact that the analogies upon which the part-whole talk is based will only extend so far. Thus we cannot say that the premisses are composed or built up from various conclusions or that the child is composed of or built up from his mother and father; and nor can we meaningfully talk of replacing the given conclusion in the premisses with another conclusion or replacing the mother in the child with someone else.

However, in the case of set-membership, there would appear to be nothing that might plausibly be taken to indicate that the talk of part-whole is not to be taken literally. A set is indeed composed of or built up of its members and we should add that we may meaningfully talk - of replacing one member of a set with another. Thus Aristotle in the set {Plato, Aristotle} may be replaced with Socrates to obtain the set {Plato, Socrates}, with the given set becoming a different set from what it was. Our conception of members as parts seems to extend all the way.

But perhaps what is most significant is that by assuming that the members are parts we can achieve a degree of generality and unity in the theory of part-whole that could not otherwise be attained. Diverse phenomena can be explained and connected under the common rubric of part-whole; and although the full force of this point will only become evident in the course of our enquiry, it is hardly good methodological practice to adopt a monist position at the start of our enquiry and thereby deprive ourselves of the possibility of discovering whether a broader conception of part-whole can be sustained. We should see if we can coherently treat members and the like as parts and only give up the assumption that they are parts if we fail.

§3 Operationalism

The adoption of pluralism has significant implications for the study of mereology. For the single way of being a part of classical mereology must now give way to a number of different ways of being a part. These should then be characterized and the connections between them explored. The adoption of pluralism will also have significant implications for metaphysics at

4Cf Lewis [1986], ref. ***
large. For if the part-theoretic structure of familiar things need not be the aggregative structure of classical mereology, then we need to investigate what it might be and how it might be realized. There is therefore the possibility of discovering new kinds of part-theoretic structure and of providing novel analyses of the part-theoretic structure of familiar things.

I am not here concerned to pursue the second line of investigation but I do wish to provide some indication of how a more general theory of part-whole might be developed. Now in formulating the principles of mereology, it has been usual to take the relation of part-whole or some associated relation (such as overlap) as primitive. But I believe that, in formulating a more general theory, it is important to take the operation of composition as primitive rather than the relation of part-whole. In the case of classical mereology, the operation of composition will take some objects into the sum or fusion of those objects, while, in the set-theoretic, it will take some objects into the set of those objects; and, in general, the operation of composition will be the characteristic means (summation, set-builder etc.) by which a given kind of whole is formed from its parts.

Even if we set philosophical considerations on one side, there are compelling logical reasons for favoring the operation over the relation. For it is always possible to define the relation in terms of the operation but not always possible to define the operation in terms of the relation. In the set-theoretic case, for example, the operation is the set-builder and the relation is ancestral membership. But it may be demonstrated to be in principle impossible to define the set-builder (or, equivalently, membership) in terms of ancestral membership. Thus someone who took the relation of part-whole to be primitive in this case would deprive themselves of the means of talking of composition.

Indeed, even in the case of classical mereology, the standard definitions of summation in terms of part-whole will only be correct under certain existential assumptions. For suppose we define $y$ to be the sum (or fusion) of $x_1, x_2, ...$ just in case (i) each of $x_1, x_2, ...$ is a part of $y$ and (ii) $y$ is a part of $z$ whenever each of $x_1, x_2, ...$ is a part of $z$ (similar considerations will apply to other definitions of this sort that might be given). Consider now an ontology consisting of three atoms $a, b$ and $c$ and a universal element $d$, as depicted below:

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        d
       / | \    
      a   b   c
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The definition would then predict that $d$ is the sum of $a$ and $b$, of $b$ and $c$, and of $a$, $b$ and $c$ when, intuitively, it is only the sum of $a$, $b$ and $c$.

Let me quickly mention a couple of other advantages that derive from adopting the operational approach, although others will become apparent in the course of the discussion. In set theory and standard mereology, there is a common notion of a ‘null’ object - where this is the null set in the case of set theory and the null sum in the case of mereology. Under the operational approach, it is possible to provide a uniform definition of the ‘null’ object, for this is the whole, if any, which results from applying the compositional operator $\Sigma$ to zero objects. But on the usual relational approach, it will be impossible to identify the null set, since its behavior with respect to membership will be indistinguishable from that of an arbitrary non-set, and it will

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5I omit the details. But let me note that the obvious definition will not work. For if one were to define $x$ to be a member of $y$ if $x$ is an ancestral member of $y$ but not an ancestral member of an ancestral member of $y$, then this would incorrectly exclude $x$ from being a member of $\{x, \{x\}\}$. 
be impossible to identify the null sum (should it exist), since it will behave in the same way as an atom that is a part of everything else. Or again, there is an intuitive distinction between wholes which are like sets in being hierarchically organized and those which are like sums in being at the same level. The distinction, under the operational approach, can be seen to turn on whether repeated applications of the operation are capable of yielding something new, whereas it is not clear what it might come to under the relational approach. That there are proper parts of proper parts is certainly not enough; and merely given the ‘graph’ of the part-whole relation, it is hard to see what else might reveal the existence of a level or hierarchical organization of the wholes.

It is important to the development of the operational approach that we take a permissive view on the form of the compositional operations. We should allow them to be variably polyadic in the sense of applying to any number of objects (including possibly none or one or infinitely many). We will want the set-builder, $\Sigma$, for example, to apply to any number of objects $x_1, x_2, \ldots$ (with $\Sigma(x_1, x_2, \ldots) = \{x_1, x_2, \ldots\}$). We should also allow the operations to be partial, i.e. not always be defined. Thus it has often been supposed that the summation operation, $\Sigma$, has no application to zero objects; and the operation for forming a predicative proposition will have no application to the Eiffel Tower and the Tower of London (since neither can play a predicative role). The operations should also be allowed to be sensitive to the order of the arguments to which they apply. The result of applying the operation for forming sequences to Socrates and Plato, for example, should be different from the result of applying it to Plato and Socrates.

With each compositional operation $\Sigma$ may be associated a corresponding relation of part. This may be defined in two steps. We first say that $x$ is a component of $y$ if $y$ is the result of applying $\Sigma$ to $x$ or to $x$ and some other objects. In other words, $y$ should be of the form $\Sigma(x_1, x_2, \ldots)$, where at least one of $x_1, x_2, \ldots$ is $x$. Thus when $\Sigma$ is mereological summation, the components of an object will be mere parts and, where $\Sigma$ is the set-builder, the components of an object will be its members. We may now define $x$ to be a part of $y$ if there is a sequence of objects $x_1, x_2, \ldots x_n$, $n > 0$, for which $x = x_1$, $y = x_n$ and $x_i$ is a component of $x_{i+1}$ for $i = 1, 2, \ldots, n-1$. The parts of an object are the object itself, or its components, or the components of the components, and so on. So whereas components will yield their wholes under a single application of the compositional operation, parts will yield their wholes under successive applications of the operation (in addition to a single application or no application at all).\(^6\)

The relation of part is plausibly taken to conform to the following three principles:

**Reflexivity** Each object is a part of itself,

**Transitivity** If $x$ is a part of $y$ and $y$ of $z$ then $x$ is a part of $x$,

**Antisymmetry** $x$ is a part of $y$ and $y$ of $x$ only when $x = y$.

These principles, or something like them, are usually taken to be axioms. But an interesting feature of the operational approach is that they can now be derived from the definition of part.

Reflexivity and transitivity follow directly from the definition, without making any assumptions about the underlying behavior of the compositional form. For the one-term chain $x_1$ (with $n = 1$) shows that $x$ is a k-part of $x$. And in regard to transitivity, let us suppose that $x$ is

\(^6\)It might be thought that some forms of composition are compositional in only some of their argument-places. The operation for forming a state from a relation $R$, some individuals $x_1, x_2, \ldots, x_n$, and the location $L$, for example, might be taken to be compositional in $R$ and $x_1, x_2, \ldots, x_n$, though not in $L$, so that the relation and the individuals would be parts of the resulting state while the location is not. In such cases, of course, the definitions of k-component and k-part should be restricted to the argument-places in question.
a k-part of y and y is a k-part of z. There will then be an appropriate chain \( x = x_1, x_2, ..., x_n = y \) of k-components connecting x to y and an appropriate chain \( y = y_1, y_2, ..., y_m = z \) of k-components connecting y to z. So their combination \( x_1, x_2, ..., x_n, y_2, ..., y_m \) will be an appropriate chain of k-components connecting x to z.

Anti-symmetry follows from the definition of part with the help of a corresponding assumption concerning the behavior of the compositional form:

**Anti-Cyclicity** If \( x = \Sigma(..., \Sigma(..., x, ... ), ... ) \), then \( x = \Sigma(..., x, ... ) \).

In other words, if x can be built up from x itself, then any intermediate whole (\( \Sigma(..., x, ... ) \)) involved in the construction must itself be x. In many cases, this assumption may be derived from more fundamental assumptions on the compositional form. When \( \Sigma \) is the set-builder, for example, we may appeal to the absence of infinitely descending membership chains to show that the antecedent \( x = \Sigma(..., \Sigma(..., x, ... ), ...) \) is never satisfied.

Although reflexivity and transitivity follow directly from the definition, there are notions of part or the like under the operational approach for which this is not so. For suppose that we take x to be a part of y if there is a sequence of objects \( x_1, x_2, ..., x_n, n > 1 \), for which \( x = x_1, y = x_n \) and \( x_i \) is a component of \( x_{i+1} \) for \( i = 1, 2, ..., n-1 \) (the case \( n = 1 \) is no longer allowed). With this notion of part in play, which is perhaps more natural than the one we previously adopted, it is a substantive question whether or not the relation is reflexive in any given case. For we are asking whether an object can be built up from itself by means of some positive application of the given form of composition; and this will depend upon how exactly the form of composition behaves. Thus when the underlying operation is summation, each object will be a part of itself since the unit sum of any object is the object itself but, when the underlying operation is the set-builder, no object will be a part of itself since no object is ever an ancestral member of itself.

Similarly, if part is understood as component, it will be a substantive question whether or not the relation is transitive. In the case of sums, for example, it will be transitive since components of components are components while, in the case of sets, it will fail to be transitive since components of components (i.e. members of members) will sometimes fail to be components (members). It is a shortcoming of the usual relational approach that it makes no room for such subtle distinctions in our understanding of part.

§4 Types of Principle

In developing a general theory of part-whole, we will wish to state the principles by which the various forms of composition are governed. If a given form of composition is definable in terms of more basic forms, we may derive the principles for the one from the principles for the others. It therefore suffices to state the principles for the most basic forms of composition, those not definable from others.

I believe that the principles governing the basic forms of composition will conform to a general template. Variations in the principles for the different forms of composition will then arise from variations in how the template is to be filled in. The template will comprise two broad categories of principle - the formal and the material. From among the formal principles, we may distinguish between those that provide conditions of application for the operation and those that provide identity conditions; and from among the material principles, we may distinguish between those that provide conditions for the presence of a whole (in space and time or at a world) and those that specify the descriptive character of the whole. The presence conditions, in their turn, may concern either the existence of the whole or its extension. The general classification of principles within the template is therefore as follows:
<table>
<thead>
<tr>
<th>Principles</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formal</strong></td>
</tr>
<tr>
<td>Application</td>
</tr>
<tr>
<td>Existence</td>
</tr>
</tbody>
</table>

Let us discuss each kind of principle in turn. The formal principles are the counterpart, within the operational approach, to the standard axioms of mereology. They can usually be stated within a purely logical vocabulary that has been enriched with whatever mereological primitive is in question. These principles are of two kinds. The first concerns the conditions under which there are wholes of a given sort - which, on the operational approach, is a matter of stating when the result of applying the compositional operation to various objects will be defined. The second concerns the conditions of identity for wholes - which, on the operational approach, is a matter of stating when a whole formed in one way by means of the compositional operation is the same as a given object or a whole that has been formed in some other way. In the case of summation, for example, it might be thought that the result of applying the sum operation to no objects is undefined (so that the null sum will not exist) and that the result of applying the sum operation to a single object is that very object.

The material principles concern the conditions under which the wholes will possess certain material features (those that are neither logical nor mereological). Two classes of material features may be distinguished: those that relate to the presence of the whole in space or time or the world; and those that concern its more descriptive character, such as its color or weight. Principles of the latter sort are usually ignored in the more formal development of mereology though they are often critical to how the subject is to be applied or put to philosophical use.

My own view is that there are two fundamentally different ways in which an object might be present in space or time; it may *exist* in space or time; or it may be *extended* or *located* in space or time. Thus a material thing will exist in time but be extended in space while an event will be extended in both space and time. This means that we will need to state separate presence conditions for the existence and extension of a whole. But someone who does not accept the distinction may simply provide conditions for a single uniform notion or presence.

The character conditions tell us what the wholes are like. In the case of volume, for example, they may tell us that the volume of a sum $\Sigma_m(x_1, x_2, ...)$ at a given time is the sum of the volumes of the components $x_1, x_2, ...$ at that time, as long as the components are spatially disjoint; or they may tell us that the sum $\Sigma_m(x_1, x_2, ...)$ is gold just in case each of the components $x_1, x_2, ...$ is gold.

The character conditions will tend to have a much more ad hoc character than the other conditions that we have considered. The color of a house, for example, is the color of its siding,  

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7See Fine ([1994a], [2006]) for a defense of this view and a consideration of some of its consequences.
the color of an egg the color of its shell, the color of a pencil the color of its lead. In so far as the
color of a whole is determined in each of these cases by a color of a part, it would appear that
there is nothing in general that we can say about when the color of a whole is so determined or
what the relevant part or parts may be, when it is. It is plausible, however, that the character of a
whole will in general be some function of the character of its parts.

I am inclined to the view that these various principles are definitive of the form of
composition. It will lie in the nature of any form of composition to conform to such principles;
such principles will be exhaustive of its nature (beyond its being a basic form of composition);
and any two basic forms of composition will differ in their nature, i.e. differ in the principles by
which they are governed. However, the character conditions may be an exception in this regard.
For it is more plausible to suppose in the case of particular material properties (such as having a
certain color) that it lies in the nature of the material properties, rather than the forms of
composition, that the appropriate conditions should hold. Thus the conditions in this case are
definitive of how the material properties are to be extended to the wholes in question rather than
of the form of composition by which the wholes are formed.

In what follows, I shall focus on the identity principles and shall largely ignore the
application and presence principles. The other principles are certainly of interest and call for an
extended discussion in their own right; and it is only for reasons of space that I do not consider
them.

§5 Mere Sums

Let us show, by way of illustration, how classical mereology can be made to conform to
the above template. This will shed some further light on what might appear to be an excessively
familiar subject, as well as helping to illustrate how our general approach is to be applied. This
case will then serve as a basis for explaining how other forms of composition naturally arise
within the operational framework.

There are two aspects to the notion of whole that have been implicit in the classical
development of mereology. The first, more formal, aspect is that a whole is a ‘mere sum’. It is
nothing over and above its parts - or perhaps we should say, more cautiously, that it is nothing
over and above its parts except in so far as it one object rather than many. The second, more
material, aspect is that wholes are ‘four-dimensional’ objects, equally extended in time as they
are in space. The two aspects usually taken together but it is possible to accept the first, and treat
the wholes as mere sums, without accepting the second, and treating them as four-dimensional
objects.

I myself favor a three-dimensional account of mere sums (treating them as ‘compounds’
rather than ‘aggregates’ in the terminology of Fine [1994a]). But in keeping with our general
focus on identity, let us put the issue of ‘dimensionality’ on one side. Our question, therefore is:
given the conception of wholes as mere sums, when should two wholes be taken to be the same?

Although the intuitive conception of a mere sum might appear to be very wishy-washy, I
believe it is possible to determine completely from this conception what the identity conditions
for them should be. An identity condition will, in general, take the form of an identity statement
flanked by terms in ∑, where these are constructed from variables and a symbol ‘∑’ for the form
of composition in question. Thus each variable x, y, z, ... will be a term; and  ∑(r, s, t, ...) will be

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8Whether we can give an explicit definition of the application of these operations is another
matter. But even if there is an explicit definition, it will not be in terms of more basic
compositional operations.
a term when r, s, t, ... are terms. Call an identity condition s = t regular if the variables of s and t are the same. We then have the following compendious statement of the identity conditions for sums:

**Summative Identity** \( s = t \) should obtain for any regular identity condition \( s = t \).

So, since the conditions \( \Sigma(x) = x \), \( \Sigma(x, y) = \Sigma(y, x) \) and \( \Sigma(x, \Sigma(y, z)) = \Sigma(\Sigma(x, z), y) \) are all regular, it will follow from the principle that they all obtain.

This principle provides formal expression of the idea that wholes built up from the same parts should be the same; and this is something that appears to be constitutive of our intuitive conception of a mere sum as nothing over and above its parts. For consider a typical case of a regular identity, say \( \Sigma(\Sigma(x, y), \Sigma(y, z)) = \Sigma(x, y, z) \). According to our intuitive conception of a mere sum, there is nothing more to the whole \( \Sigma(\Sigma(x, y), \Sigma(y, z)) \) than its parts \( \Sigma(x, y), \Sigma(y, z) \) and nothing more to the wholes \( \Sigma(x, y), \Sigma(y, z) \) than their parts \( x, y \) and \( y, z \) respectively. Presumably, it follows from this that there is nothing more to \( \Sigma(\Sigma(x, y), \Sigma(y, z)) \) than \( x, y \) and \( y, z \) and that, likewise, there is nothing more to the whole \( \Sigma(x, y, z) \) than its parts \( x, y, \) and \( z \). But if there is nothing more to either whole than the parts \( x, y, \) and \( z \), then it presumably follows that the two wholes should be the same. Thus philosophical reflection on the notion of a mere sum is able to provide us with a simple and natural characterization of classical mereology.

But simple as this characterization may be, the set of identities that it appeals to is highly redundant. From \( \Sigma(x) = x \), for example, it will follow that \( \Sigma(\Sigma(x)) = \Sigma(x) \) and hence that \( \Sigma(\Sigma(x)) = x \); and there is therefore no need to appeal to all of these identities. A more compact formulation may be obtained by providing an analysis of the different grounds upon which a regular identity may hold.

We thereby arrive at the following four principles:

- **Absorption** \( \Sigma(..., x, x, ..., y, y, ..., ...) = \Sigma(..., x, ..., y, ...) \);
- **Collapse** \( \Sigma(x) = x \);
- **Leveling** \( \Sigma(..., \Sigma(x, y, z, ...), ..., \Sigma(u, v, w, ...), ...) = \Sigma(..., x, y, z, ..., u, v, w, ..., ...) \);
- **Permutation** \( \Sigma(x, y, z, ...) = \Sigma(y, z, x, ...) \) (and similarly for all other permutations).

According to Absorption, the repetition of components is irrelevant to the identity of the whole; according to Collapse, the whole composed of a single component is that very component; according to Leveling, the embedding of components is irrelevant to the identity of the whole; and according to Permutation, the order of the components is irrelevant to the identity of the whole. These four principles follow from Summative Identity; and, conversely, Summative Identity follows from them. Thus together they constitute an analysis of the notion of a mere sum.
§6 More-than-mere Sums

The previous principles - Absorption, Collapse, Leveling and Permutation - point to different features of composition; and this suggests that there may be forms of composition that satisfy some of these principles but not others. Indeed, I think it will be found that for many subsets of these principles, there will naturally correspond a form of composition - sometimes familiar and sometimes not so familiar - that satisfies each of the principles of the subset and fails to satisfy the others.

Let us go through the cases in turn, using the letters A, C, L and P as a mnemonic for the respective principles. CA, for example, will be used for a form of composition that satisfies C and A but not L or P. There are four cases that will serve as familiar points of reference:

- CLAP: Sums.
- AP: Sets. Repetition and order of components are irrelevant.
- CL: Strings. The form of composition is concatenation and concatenating two strings, say xy and uv, is the same as concatenating their components x, y, u and v.
- : Sequences. The form of composition is the 'sequence-builder', where sequencing two sequences (xy) and (uv) to obtain ((xy)(uv)) is to be distinguished from sequencing x, y, u and v to obtain (xyuv) (in contrast to the case of strings).

These four cases are determined by the natural association between C and L and between A and P; either one of each pair is always accompanied by the other. Thus the possibilities may be set out in the following chart:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>sums</strong></td>
<td><strong>sets</strong></td>
<td></td>
</tr>
<tr>
<td><strong>strings</strong></td>
<td><strong>sequences</strong></td>
<td></td>
</tr>
</tbody>
</table>

where the ‘horizontals’ (sums and sets or strings and sequences) and the ‘verticals’ (sums and strings or sets and sequences) represent axes of similarity and where the ‘diagonals’ (sums and sequences and strings and sets) are most opposed.

From each of the pairs (C, L) and (A, P), L or P is the ‘dominant’ or more significant element; and so we may say that a form of composition is sum-like if it conforms to LP, set-like if it conforms to P, string-like if it conforms to L; and sequence-like if it conforms to . Thus within each of these categories there are three possible variants on the core member. Thus in addition to sets, the category of set-like forms of composition will include:

- CAP: Quinean sets - like sets but with the singleton identical to its sole component;
- P: Multi-sets - like sets but sensitive to multiple occurrences of the same component.

Thus the multi-sets [x] and [x, x] are to be distinguished, with the first containing one occurrence of x and the second containing two occurrences of x.

- CP: Quinean multi-sets - like multi-sets but with the singleton identical to is sole component.

And similarly for the other categories.\textsuperscript{13}

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12The above formulation presupposes that the sum $\sum(x, y, z, ...)$ is always defined. We may avoid making this presupposition by stating the principles as ‘weak’ identities, whose truth only requires that the terms on the right and left be co-designative when one or the other of them is defined.
13Though from among the twelve possible variants, two (ACL and AL) should not be allowed since they are in violation of Anti-cyclicity.
The sense in which these various forms of composition will fail to satisfy the relevant principles calls for further comment. It is not enough to say that the principle *sometimes* fails. For then a single case in which \( \{x\} \neq x \) fails would enable the set-builder to fail to satisfy Collapse. And it would be going too far to say that the principle *always* fails. After all, the string \( xy \) will be the same as the string \( yx \) when \( x \) and \( y \) are the same even though concatenation fails to satisfy Permutation. Rather the only identities that should hold are the ones that can be shown to hold on the basis of the defining principles. Thus it is because there is no way to establish \( \{x\} = x \) on the basis of Absorption and Permutation, no matter how the object \( x \) might be designated, that \( \{x\} = x \) will always fail; and, as a rule, different designations of objects will designate different objects unless the designations can be shown to be co-designative on the basis of the identity principles.

The above approach provides a unified treatment of various familiar operations that might have been thought to be unconnected under the relational approach; and it points to the existence of a variety of other operations that might otherwise have gone unnoticed. Indeed, there would appear to be no good reason to require that the defining principles for the various operations be limited to the particular principles \( (C, L, A \text{ and } P) \) that we used in characterizing sums; for *any* set of regular identities would appear to be equally well suited to defining a basic form of composition, so long as they conform to Anti-cyclicity. Thus the operational characterization of summation naturally leads us to suppose that mereological monism must be false, since any other characterization of this sort would appear to be equally capable of giving rise to a compositional form.

Vast as the ensuing forms of composition may be, they do not by any means exhaust the basic forms of composition that there are. They do not include the operation of predication, for example, whereby the subject-predicate proposition that \( x \) F’s is formed from a predicative component \( F \) and a subject component \( x \); and nor do they not include any other form of composition in which some of the components are required to play a distinctive ‘unifying’ role. Even in these other cases, we would hope to be able to characterize the form of composition by means of the principles to which it conforms; and it is a large and difficult question, on which we have only made a start, to say which sets of principles are capable, in general, of characterizing a basic form of composition.

§7 Derived Part

It is plausible to suppose that every form of composition is either basic or to be derived from the basic forms of composition and that every relation of part is either basic or to be derived from the basic relations of parts. But what are these derivative forms of composition or relations of part and how are they to be derived from the basic forms and relations?

There are two obvious routes by which derivative relations of part may arise. One is through Subsumption and is illustrated by the case of *small part*. When \( x \) is a small part of \( y \), then this is something that holds, at least in part, in virtue of a more basic relationship of part-whole, viz. that \( x \) is a part of \( y \).

The other route is through Chaining. Suppose, for the sake of illustration, that a pint of milk is a part of a gallon of milk in one way and that Socrates is a part of singleton Socrates in another way. There would then be a way in which the pint of milk is a part of the singleton

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14Or to put it algebraically, the intended model for the principles should be isomorphic to a ‘free algebra’ over the ‘generators’ or given elements. Some identities among the generators might be allowed; and the intended model should then be isomorphic to the corresponding word algebra.
gallon of milk. But this is a relationship of part that would hold, at least in part, in virtue of the pint being a part of the gallon (in one way) and the gallon being a part of the singleton gallon (in another way). The given relationship of part is mediated, so to speak, though these other relationships of part.\footnote{5} 

Similarly, in the case of composition, Subsumption or Chaining provide the two obvious routes by which one form of composition might be derived from others. Thus the operation of summation might be restricted to material things or to events; or we might restrict the application of the set-builder to one or two objects and then chain it to obtain the operation that yields the standard set-theoretic representation $\{\{x\}, \{x, y\}\}$ of the ordered pair from the component terms $x$ and $y$.

Are these the only means by which the non-basic relations of part or forms of composition can be derived from the basic relations or forms? I believe that the answer to this question is ‘yes’ in the case of part but that, somewhat surprisingly, it is ‘no’ in the case of composition. There are, it seems to me, other, more ‘creative’, ways in which the basic forms of composition may give rise to other forms of composition.

For consider the case of sets. The union $(x_1, x_2, ...)$ = $x_1 \cup x_2 \cup ...$ of the sets $x_1, x_2, ...$ is intuitively a way of composing a whole from its parts. But the application of this operation is not something that holds via the subsumption or chaining of other operations. When $y = (x_1, x_2, ...)$ it is not as if $y$ is the set of $x_1, x_2, ...$ and nor do there appear to be any other more basic compositional operations through which $y$ might be obtained from $x_1, x_2, ...$. Similarly for the union of multi-sets (whereby $[x, x, y] \cup [x, y]$, say, is $[x, x, y, y]$) or for the juxtaposition of sequences (whereby $<a, b> \ast <c, d, e>$, say, is $<a, b, c, d, e>$). They intuitively provide us with a way of composing a whole from its parts, even though their application cannot be subsumed under the multi-set-builder or sequence-builder or obtained, in some other way, through the subsumption or chaining of more basic operations.

Within the operational framework, we can provide a general account of how these new forms of composition arise. Using the case of sets as an example, we may provide the following definition of the operation of set-theoretic union in terms of the set-builder:

\[
\text{from } \{\ldots\} \ (\{x_{11}, x_{12}, \ldots\}, \{x_{21}, x_{22}, \ldots\}, \ldots) = \{x_{11}, x_{12}, \ldots, x_{21}, x_{22}, \ldots, \ldots\}.
\]

In other words, the union operation, in application to given sets, will result in the set which is built up from the objects from which the given sets were built up.

This suggests a general recipe for defining a ‘horizontal’ operation in terms of a given ‘vertical’ operation. If $\sum$ is the vertical operation, the corresponding horizontal operation $\sum$ is to be defined by:

\[
\sum \text{ from } \sum \sum \sum \sum \sum \sum \sum (x_{11}, x_{12}, \ldots), \sum(x_{21}, x_{22}, \ldots), \ldots = \sum(x_{11}, x_{12}, \ldots, x_{21}, x_{22}, \ldots, \ldots).
\]

So, for example, if $\sum$ is the sequence-builder $<\ldots>$, then the corresponding operation $\sum$ of juxtaposition will be defined by:

\[
\sum <\ldots>(<x_{11}, x_{12}, \ldots>), <x_{21}, x_{22}, \ldots>, \ldots = <x_{11}, x_{12}, \ldots, x_{21}, x_{22}, \ldots, \ldots>.
\]

\footnote{15}Since any relation of part must be transitive, it is only the \textit{ancestral} of the restriction or the chaining of part-whole relations that can strictly be said to be a relation of part.

\footnote{16}Of course, the acceptability of this definition depends upon its not being sensitive to the way the $\sum$-wholes are represented on the left: if $\sum(x_{11}, x_{12}, \ldots) = \sum(y_{11}, y_{12}, \ldots)$, $\sum(x_{21}, x_{22}, \ldots) = \sum(y_{21}, y_{22}, \ldots)$, ..., then $\sum(x_{11}, x_{12}, \ldots, x_{21}, x_{22}, \ldots, \ldots) = \sum(y_{11}, y_{12}, \ldots, y_{21}, y_{22}, \ldots, \ldots)$. I have assumed, for simplicity, that the relevant $\sum$-terms are always defined. The account is readily modified to take care of the cases in which this assumption fails; and there are some other variants on the method that I have also not discussed.
I am inclined to believe that this method for deriving forms of composition is of general application and that the method, in conjunction with Subsumption and Chaining, is capable of providing all the means by which one form of composition may be derived from others.

The new forms of composition will give rise in the usual way to corresponding relations of part-whole. Thus corresponding to set-theoretic union is the relation of set-inclusion, corresponding to multi-set union is the relation of multi-set-inclusion, and corresponding to juxtaposition is the relation of subsequence. Even though the horizontal operation will be definable in terms of the vertical operation (set-theoretic union in terms of the set-builder, for example), it will not in general be possible to define the corresponding horizontal relation of part in terms of the corresponding vertical relation (set-theoretic union in terms of the ancestral of membership). We cannot define x to be a subset of y when every ancestral member of x is an ancestral member of y, for example, since then \{x, \{x\}\} would be a subset of \{\{x\}\}; and nor is any other definition available. We should therefore recognize that there are basic relations of part that do not correspond to basic forms of composition; and this provides yet another reason for adopting the operational approach, since it is only at the level of the operations that the relationships between these different notions of part can be made clear.

§8 Hybrid and General Part

Given the specific relations of part, we may derive various hybrid relations of part. Suppose, for example, that we are given the relations of set-theoretic and mereological part - which we may designate as ε-part and and m-part. We may then take an one object to be an ε,m-part of another if it is an ε-part or an m-part or an m-part of an ε-part or an ε-part of an m-part, or an m-part of an ε-part of an m-part, and so on. More generally, if K is a family of specific ways of being a part, we may take an object to be a K-part of another if x and y can be linked by relationships of k-part for k in K.

The hybrid relationships of part often appear to have an artificial character, as if there were something illegitimate in passing from the one way of being a part to the other. Thus even though the letter ‘i’ is a part of the word ‘it’ and the raised dot ‘.’ is a part of the letter ‘i’, it is odd to say the raised dot is a part of the word ‘it’. Likewise, even though a gallon of milk g may be part of the singleton of the gallon and a pint of milk may be a part of the gallon, it is odd to say that the pint is a part of the singleton.

The oddity of these cases might be used as an additional test for distinguishing between the different specific ways of being a part. For if the pint were a part of the gallon in the same way in which the gallon was a part of the singleton, there should be no oddity in saying that the pint was a part of the singleton. However, I do not think that the oddity of these cases should lead one to deny that the relevant objects are parts. For if the pint is in the gallon, in the sense of ‘in’ appropriate to part, and the gallon is in the singleton, then it is hard to see how the pint could fail to be in the singleton.

To each family of specific ways of being a part will also correspond a family of compositional forms; and just as we may attempt to characterize the principles by which the specific compositional forms are governed, so we may attempt to characterize the principles by which they interact. How for example do sets interact with sequences or sets with sums or all three interact with one another? And in this regard, the critical question concerns inter-species identity. When is a whole of one kind identical to a whole of some other kind?

My inclination here is to suppose, as before, that the only identities that hold are the ones that can be shown to hold on the basis of the defining principles for the operations in question.
So, since the operation for forming sums is subject to Collapse, the unit sum of a set must be identical to that very set but, since there is nothing in the identity principles governing sums and sets that would force the sum of two or more sets to be identical to any set, no sum of two or more sets will ever be a set.\footnote{Again, this idea may be made precise by appeal to the appropriate word algebra.} This conclusion is contrary to the view of Lewis [1991], who takes each set to be the sum of its singleton subsets, but Lewis’s view is based more upon the desire to reduce set theory to mereology than to understand the actual mereological structure of sets.

Among the hybrid relations of part, of special interest is the relation of K-part where K is the family of all the specific ways of being a part. This is the relation of K-part that holds between two objects when they may be linked by relationships of k-part without restriction on k. We might call it the general relation of part and it is a relation that holds between x and y whenever x is in any way a part of y.

The general relation, through its transitive application, provides us with a highly artificial sense of part; and it is not clear that we would ordinarily have a use for a notion of such wide generality, since our interest is usually in some specific relation of part or in some small family of closely connected relations of part. However, the problem of establishing that the general relation is antisymmetric is of enormous technical and philosophical interest.

It seems clear, in the first place, that the general relation should be anti-symmetric (thinking of it now as the result of chaining the various specific relations of part and not necessarily as a relation of part in its own right). For suppose that x is a part of y in some specific way but distinct from y. There would then appear to be a broad sense of ‘more’ in which it is correct to say that there is more to y than x. Suppose now that x\(_1\) is a part of x\(_2\) in some specific way, x\(_2\) a part of x\(_3\) in some specific way, ..., and x\(_n-1\) a part of x\(_n\) in some specific way, where each of x\(_1\) and x\(_2\), x\(_2\) and x\(_3\), ..., and x\(_n-1\) and x\(_n\) are distinct. There should then be more to x\(_2\) than x\(_1\), more to x\(_3\) than x\(_2\), ..., and more to x\(_n\) than x\(_n-1\) and hence more to x\(_n\) than x\(_1\). Hence x\(_1\) and x\(_n\) in such a sequence cannot be the same - which is just what is required for the general relation to be anti-symmetric.

However, anti-symmetry is not simply one condition among others that the general relation of part should satisfy. It provides a key test for our having a coherent conception of part in the first place. For under the pluralist approach, we have wished to maintain that there are many different ways in which one object can be part of another (through membership, subset, mere part etc.). But what assurance can we have that our judgements in all of these cases are informed by a single coherent conception of part? A key - perhaps the key - test of coherence is that the resulting general relation of part should be anti-symmetric. For it would be too much of a coincidence, so to speak, if anti-symmetry held even though there was no single coherent conception of part in virtue of which it could be seen to hold.

Despite its importance, it is far from trivial to establish that the test is met. Indeed, as we have already seen, it is not even straightforward, once we adopt the operational approach, to show that the basic relations of part-whole are antisymmetric, since this must be shown by appeal to the underlying properties of the compositional form. But even if the basic relations of part are assumed to be anti-symmetric, there is still no assurance or obvious way to establish that the general relation of part will be anti-symmetric. For what is to stop one relation of part-whole taking us from x to a distinct object y and some other relation or relations of part-whole taking us back from y to x?

We may illustrate in miniature the nature of the difficulty by considering the case of membership and subset. It is straightforward to show that each of these relations is anti-
symmetric, the first because we never have infinitely descending membership chains, with \( x_2 \in x_1, x_3 \in x_1, \ldots \), and the second because mutual subsets will have the same members and will therefore be the same sets. But what of the hybrid relation, in which we may arbitrarily ‘chain’ membership with subset? It is also anti-symmetric, though it is not altogether straightforward to show that this is so.\(^{18}\)

This is but one case; and what we require is a completely general proof of anti-symmetry, one that will cover all of the different cases that might arise.

§9  Priority and Generation

One object may be (ontologically) prior to another in the sense that it is possible to provide an explanation of the identity of the one object, to explain what it is, in terms of the other object. Thus it is plausible to suppose that the members of a set are prior to the set, since one may account for the identity of the set by saying that it is the set with these members; and it is plausible to suppose that the two pints of milk that make up a quart of milk are prior to the quart, since one may account for the identity of the quart by saying that it is the quantity of milk that is made up of those two pints.\(^{19}\)

In many cases, including all those of interest to us, an explanation of identity will proceed via the application of an appropriate operation. In other words, there will be an operation - taking objects into objects - and the explanation of the identity of a particular object is that it is the result of applying this operation to certain other objects. The particular object is the result of applying the operation, not just in the innocuous sense of being identical to the result, but also in the philosophically significant sense of having its identity thereby explained. So, for example, we may account for the identity of the set \{Socrates, Plato\} by saying that it is the result of applying the set-builder to Socrates and Plato and we may account for the identity of the proposition that Socrates is wise by saying that it is the result of applying the operation of predication to Socrates and the property of being wise.\(^{20}\)

We might say that the application \( y = \Gamma(x_1, x_2, x_3, \ldots) \) of a basic operation \( \Gamma \) is generative if there is a (weak) explanation of the identity of \( y \) as \( \Gamma(x_1, x_2, x_3, \ldots) \); and we might say that the operation \( \Gamma \) is itself generative if it permits a generative application. Thus both the set-builder and the operation of predication will be generative in this sense.

It should not be thought that every application of a generative operation will itself be generative. Consider the operation for forming sums. I take it that we can account for the identity of the sum of Socrates and Plato in terms of its being the result of applying this operation to Socrates and Plato. So the operation is indeed generative. However, we cannot account for the application \( y = \Gamma(x_1, x_2, x_3, \ldots) \) of a basic operation \( \Gamma \) is generative if there is a (weak) explanation of the identity of \( y \) as \( \Gamma(x_1, x_2, x_3, \ldots) \); and we might say that the operation \( \Gamma \) is itself generative if it permits a generative application. Thus both the set-builder and the operation of predication will be generative in this sense.

\(^{18}\)Here is one possible method of proof. Let \( TC(x) \) be the transitive closure of \( x \). Show: (i) \( x \in y \rightarrow TC(x) \subseteq TC(y) \); and (ii) \( x \subseteq y \rightarrow TC(x) \subseteq TC(y) \). Consider now a sequence \( x_1, x_2, \ldots, x_n \), in which \( x_i \subseteq x_{i+1} \) or \( x_i \not\subseteq x_{i+1} \) for \( i = 1, 2, \ldots, n-1 \). Then either the latter case always holds, in which case \( x_1 \subseteq x_n \) and \( x_1 \neq x_n \), or the former case sometimes holds, in which case it follows by (i) and (ii) that \( TC(x_i) \subseteq TC(x_n) \) and so, again, \( x_1 \neq x_n \).

\(^{19}\)For some purposes, we need to distinguish between weak and strong priority. If gunk is basic, then the disjoint quantities of gunk \( g_1 \) and \( g_2 \) will be weakly prior to their sum \( g = g_1 + g_2 \), since \( g \), \( g_1 \), and \( g_2 \) are on the same ontological level, while, if atoms are basic, then the distinct atoms \( a_1 \) and \( a_2 \) will be strictly prior to their sum \( b = a_1 + a_2 \), since \( b \) is on a higher ontological level than \( a_1 \) or \( a_2 \).

\(^{20}\)These ideas are meant to relate, of course, to the essentialist framework of Fine ([1994b], [1995a], [1995b]) though, in the present context, they are capable of standing on their own.
the identity of Socrates in terms of his being the result of applying this operation to Socrates himself, since the purported explanation of identity is circular. We shall later give some less obvious examples of how circularity can arise.

We might take the generative core of a generative operation to be the result of restricting the operation to its generative applications. Given the generative core $\Gamma^*$ of a generative operation $\Gamma$, we might define the notion of (weak) priority in broad analogy to the earlier definition of part. Thus we may say that $x$ is a prior component of $y$ if $y$ is the result of applying $\Gamma^*$ to some objects that include $x$ - i.e. if $y$ is of the form $\Gamma^*(x_1, x_2, ...)$, where $x$ is a among the objects $x_1, x_2, ...$. We may now define $x$ to be prior to $y$ if there is a sequence of objects $x_1, x_2, ..., x_n$, $n > 1$, for which $x = x_1$, $y = x_n$ and $x_i$ is a prior component of $x_{i+1}$ for $i = 1, 2, ..., n-1$. Thus analogously to the case of part, prior components yield their posterior object under a single application of the generative operation, while priors also yield their posterior under successive applications of the operation. The above definitions are all relative to the given generative operation $\Gamma$. But we may obtain a general notion of priority by allowing the generative operation to vary from one step in the chain $x_1, x_2, ..., x_n$ to the other.

I have so far given a philosophical characterization of the generative core $\Gamma^*$, but I believe that it is also possible to give a mathematical characterization, i.e. to give a mathematical definition of $\Gamma^*$ in terms of $\Gamma$. The central idea is to think of the objects in question as being introduced in stages through the application of $\Gamma$ to some given objects. Thus we start with givens; we may then introduce further objects through the application of $\Gamma$ to the givens, then further objects still through the application of $\Gamma$ to the objects so far introduced, and so on. A generative application of $\Gamma$ to $x_1, x_2, ...$ is then an application that can properly be used to introduce the object $y = \Gamma(x_1, x_2, ...)$. The idea is, of course, already familiar from the cumulative hierarchy in set theory. But whereas the introduction of sets is incapable of giving rise to cycles (with $x$ being introduced from $y$, for example, and $y$ from $x$), this possibility cannot be excluded in the more general case of generative operations; and in order to avoid cycles, we explicitly disallow an object to be introduced on the basis of another object if the other can equally well be introduced on the basis of it.21

We see that the generative applications of a generative operation are generative in two senses of the term. For one can explain the identity of the generated object in terms of the objects from which it is generated and one can introduce the object into the ontology on the basis of the objects from which it is generated. Though distinct, the two senses are related, since the possibilities for introducing an object into the ontology are symptomatic of how its identity is to be explained. For us, therefore, the idea of generation is not simply a metaphor but has genuine ontological bite.

§10 The Generation of Parts

Every basic compositional operation is plausibly taken to be generative. For how could there be a basic way of forming wholes unless it were sometimes explanatory of the identity of the whole that was thereby formed? It is somewhat questionable whether any basic generative operation is also compositional and, indeed, it seems to me that some basic generative operations are de-compositional. Far from serving to account for the identity of a whole in terms of its

21 We must also take account of rank, i.e. of the first ordinal stage at which an object can be introduced. A generative application must not then take us from objects of higher rank to an object of lower rank.
parts, they serve to account for the parts in terms of the whole.

One example is that of a segment or cross-section of a material thing. Let us suppose that the universe consists of atoms which are physically indivisible but of finite volume. We might then distinguish between the upper and lower parts of the atom (relative to its orientation at a given time); and it is plausible that the atoms are givens, there being no explanation of their identity in more basic terms, while the identity of the upper and lower parts of an atom is to be explained in terms of their being the upper and lower parts of the atom. Thus the account of the part is in terms of the whole rather than the other way round.

This suggests that there is a generative operation $\mathcal{I}$, of segmentation which, in application to a material thing $x$ and a spatio-temporal extension $R$, will result in the restriction $x/R$ of the object $x$ to $R$ (assuming that there is such an object). Segmentation generalizes the familiar idea of a time-slice or temporal part, the temporal parts of a thing being the special case in which it is restricted to an instantaneous slice of space-time.

Just as with the compositional operations, the present operation might be taken to be defined by various principles to which it conforms. Let us use $|x|$ for the spatio-temporal extension of $x$ and let us talk of the union or intersection of space-time regions in the obvious way. These principles might then be taken to include:

(i) there is an object $x/R$ when $|x| \cap R$ is non-empty;
(ii) $x = x/V$ (where $V$ is the whole of space-time);
(iii) $x/R = x/S$ if $|x| \cap R = |x| \cap S$;
(iv) $|x/R| = |x| \cap R$;
(v) $\sum_{m} (x/R_{1}, x/R_{2}, ...) = x/(R_{1} \cup R_{2} \cup ...)$.  

Clause (i) is an application condition; it states that there will be a segment $x/R$ if the restrictor $R$ overlaps in extension with the parent object $x$. Clauses (ii) and (iii) are identity conditions: the first states that the ‘degenerate’ restriction of $x$ to the whole of space-time is $x$ itself; and the second states that two restrictions of an object are the same if they result in the same restriction on its extension. Clause (iv) is a presence condition; it states that the extension of a restriction is the intersection of the extension of the parent object with the restrictor.

Clause (v) is something new; it states that the sum of the restrictions of a particular object is the restriction of the object to the union of its restrictors. It is this principle which conveys the distinctive mereological character of segments. In contrast to the previous defining principles, it makes reference to another generative operation. Thus there is a sense in which the operation of segmentation is not autonomous and must be understood by reference to the corresponding compositional operation. I suspect that the same is true of any other ‘anti-compositional’ operation and that the general conception of a whole must always have priority, in this sense, over the general conception of a part.

It is through the application of the operation of segmentation that we may explain how it is that a thing may be prior to its segments. For if we start off with the givens (the atoms, say), then the operation of summation will only take us so far in generating the objects of the ontology. We may obtain sums of atoms, but not segments of atoms or sums of segments or segments of sums. It is only through the application of segmentation that these other objects can be introduced into the ontology; and the resulting segments must therefore be posterior to the larger ‘wholes’ from which they were obtained.

This is in contrast to the usual picture, which sees the ontology of wholes and parts in flat

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22For simplicity, I have ignored the modal aspect of objects and I have also ignored the complications which arise from distinguishing between existence and extension.
terms, without regard for how it might be generated. But once one grants that an ontology might be generated by its atoms, with every object having its ‘source’ or ‘identity’ in the atoms by which it is composed, then one should be equally open to the possibility that the generation of objects might proceed in the other direction. One therefore arrives at the present view, in which both summation and segmentation can serve as means by which one object is generated from another.

Another significant example of a decompositional operation is given by abstraction. Consider a complex proposition, such as the proposition that Socrates is wise and Socrates is a philosopher (Ws & Ps). Then the natural analysis of this into parts is as the conjunction of the proposition that Socrates is wise and the proposition that Socrates is a philosopher, where these propositions, in turn, are the result of predicating the respective properties of being wise and being a philosopher of Socrates. But there would appear to be another analysis of the proposition into parts, whereby it is the result of predicating the complex property of being a wise philosopher (λx(Wx & Px)) of Socrates. Appeal to the complex property is not essential to analyzing the given proposition. But it may well be essential to analyzing a quantified proposition, such as that someone is wise philosopher (∃x(Wx & Px)); and so we should recognize that there are such properties, if we are to give an account of all the propositions that there are.

What account can we give of the identity of the complex property? One possibility is to treat it as the conjunction of the properties of being wise (λxWx) and of being a philosopher (λxPx). But such an approach, when applied across the board, would lead to an unwieldy parallel treatment of the various logical operations, such as conjunction or quantification, and might not even be workable in certain cases. A more promising approach, to my mind, is to treat the complex property of being a wise philosopher as the result of abstracting the individual Socrates from the proposition that Socrates is a wise philosopher. Thus we start with the proposition that Socrates is a wise philosopher (or with some other individual in place of Socrates) and then take the property of being a wise philosopher to be the result of ‘removing’ or abstracting that individual from the proposition.

In general, we may suppose that there is a generative operation, Λ, of abstraction which, in application to a complex C and an individual x, will result in the abstract [ΛxC] that results from removing the individual x from the complex C. As with segmentation, we may take Λ to be ‘defined’ by means of an appropriate set of principles. Let us use the notation ‘C(x)’ for the corresponding operation of concretion, whereby C(x) is the result of ‘completing’ C with x (predication corresponding to the case in which C is a property). We would then expect to have the two operations related by the following fundamental principle:

[ΛxC](x) = C,

according to which re-concretizing an abstraction on a complex will return us to the complex. Given that concretion is a compositional operation, it will follow that the result of abstracting on a complex is indeed a part of the complex.

It is again by reference to the generational framework that we may explain how complex properties or abstracts are posterior to the propositions or complexes to which they belong.
if we just start off with the basic ingredients from which propositions are constructed (simple properties, individuals etc), we will not be able to obtain complex properties or the quantified propositions that are constructed from them. It is only through the application of abstraction or the like that these other objects can be introduced into the ontology; and the resulting abstracts must therefore be posterior to the complexes from which they were obtained and of which they are a part.

Although the operations of segmentation and abstraction appear to be completely different and to have application in completely different spheres, we see from our analysis that there is a very deep analogy between them. For just as segmentation ‘manufactures’ parts which can be put together by the operation of summation to give us back the object from which they were obtained, so abstraction will ‘manufacture’ parts which can be put together by the operation of concretion (or predication) to give us back the complexes (or propositions) from which they were obtained. And this suggests that there should be other pairs of operations which behave in a similar way and thereby give rise to other forms of generated part.

The theory of generation enables us to ‘round out’ the theory of part in a way that would not otherwise be possible; and let me conclude by briefly discussing two key respects in which this is so. In the first place, it is natural to suppose that wholes may be classified into kinds - into sums, sets, subject-predicate propositions etc. Now the obvious definition of a kind of whole on the pure operational approach is that an object will be of a given kind k if it the result \( \sum_k(x_1, x_2, \ldots) \) of applying the associated operation \( \Sigma_k \) to a number of objects \( x_1, x_2, \ldots \). But then any object must be classified as a sum, since the unit sum \( \Sigma_m(x) \) of any object \( x \) is that very object; and this is clearly not our intention. To get round this difficulty, we might take a whole of kind \( k \) to be an object that is the result \( \Sigma_k(x_1, x_2, \ldots) \) of applying the operation \( \Sigma_k \) to a number of objects \( x_1, x_2, \ldots \), one of which is not identical to \( x \). But the null set would not then be a set-theoretic whole on this definition and, more seriously, each atom, from our earlier example, would be a sum, even though we would not wish to classify it as a sum, i.e. as something that was by its very nature a sum.

We may appeal to the theory of generation to solve this difficulty. For a genuine whole of a given kind \( k \) may be taken to be an object that has an explanation of its identity in terms of the associated operator \( \Sigma_k \). In other words, \( x \) should be identical to \( \Sigma_k(x_1, x_2, \ldots) \) for some generative application of \( \Sigma_k \) to \( x_1, x_2, \ldots \). This definition avoids the previous difficulties, for the null set would be obtained through a generative application \( \Sigma_e() \) of \( \Sigma_e \) to zero objects and so would be classified as a set-theoretic whole, while the unit sum \( \Sigma_m(x) \) and the sum \( \Sigma_m(a_1, a_2) \) of the segments \( a_1 \) and \( a_2 \) of an atom \( a \) would not be obtained through generative applications of \( \Sigma_m \) and so would not be classified as sums.

In the second place, the admission of generated parts makes it much more difficult to show that the general relation of part is anti-symmetric. For wholes are not simply ‘built up’ from their parts, parts are also ‘built down’ from their wholes; and so we must now envisage the possibility that an object \( y \) might be built up into the distinct object \( x \) and built down into \( x \), thereby creating a violation of anti-symmetry.

I believe that it is only by looking at the way in which the generation of parts and wholes might be constrained that we are able to show that possibilities of this sort cannot arise. Thus even though the anti-symmetry is a thesis lying purely within the theory of part, it is only by appeal to the theory of generation that it can be shown to be true.

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25Van Inwagen [2006] denies that there is a nontrivial kind, sum. But if I am right, he misconstrues the relationship between sum and sum of.
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