On the alleged innocence of mereology

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0. Introduction

(1) There is a cat, Mina, which is sleeping.
(2) There is a mouse, Gino, which is dancing.

The goal of this paper is to analyse arguments pro and cons the ontological innocence of mereology. We argue that:

(T1) arguments for the ontological innocence of mereology are not conclusive.
(T2) Some arguments against the ontological innocence of mereology show some ambiguity in the thesis itself.
(T3) The innocence thesis, apart from Lewis' defence, seems to depend on a general conception of the nature of objects and on how the notion of ontological commitment is understood. Specifically, we think that the thesis is the manifesto of a realist conception of parts and sums.
(T4) The alleged innocence of mereology is subject to Quine's notorious criticisms of the set-theoretical interpretation of second order logic. To the purpose, we construct a mereological model of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic.

The paper is divided into six sections. In the first one we recapitulate Lewis' version of mereology. In the second section we analyze Lewis' argument for the innocence of mereology: an argument grounded on the thesis of Composition as identity. Lewis analyses two different versions of the thesis: the first one is the Strong composition thesis (StrongCom), according to which certain objects are their sum, when the use of "are" would mean that composition is literally identity. The second version is the Weak composition thesis (WeakCom), according to which composition is similar or analogous, under some aspects, to identity. In the third section we analyze some arguments pro and cons (StrongCom), specifically Lewis, and Van Inwagen arguments against (StrongCom), and we argue for (T1) and (T2). In the fourth section we analyse arguments pro and cons (WeakCom). Specifically, we analyse arguments pro (WeakCom) given by Lewis. Again, some arguments pro (T1) and (T2) are given in this section. In section five we construct a mereological model of a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic, and we argue for (T4). In the last section (6) we give an argument for (T3).

1. Lewis treats mereology in a plural language, a language extending that of first logic, including singular and plural reference, singular and plural quantification. In such a language we consider logical terms:

(a) Plural terms, for example the pronoun "them", or plural variables, for example "X" as symbolic counterpart.
(b) Plural quantifiers, for example, "there are some things... such that".
(c) A special two-place predicate "... is one of...": This predicate admits a singular term in its first place and a plural one in its second place.

(Def.1) y and x overlap if and only if there is a z such that it is part of x and part of y.
(Def.2) y is a sum of the X if and only if each of the X is a part of y and each part of y overlaps one of the X.
(Def.3) The X compose y if and only if y is the sum of the X.
(Def.5) x and y are disjointed if and only if they do not overlap.
(Reflexivity) x is part (non proper part) of itself.
(Transitivity) If x is part of some part of y, then x is part of y.
(Unrestricted Composition) If there are some X there is a sum of the X.
(Uniqueness of Composition) If y and z are sums of the same X then y = z.
(Theorem) If there are two objects, neither of which is part of the other, then there is something else that is not identical with either of them.

2. Lewis' argument for the innocence of mereology.

(3) There is a sum of the mouse Gino and the cat Mina, Gina.

"Given a prior commitment to cats, say, a commitment to cat-sums is not a further commitment. The sum is nothing over and above the cats that compose it. It is just them. They just are. Take them together or take them separately, the cats are the same portion of Reality either way. Commit yourself to their existence all together or at one time, it is the same commitment either way... I say that composition... is like identity. The 'are' of composition is, so to speak the plural form of the 'is' of identity. Call this the Thesis of Composition as Identity. It is in virtue of this thesis that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before" [Lewis 1981: 81-82].

(P1) Composition – a many-one relation – is like identity.
(P2) The commitment to sums is already presupposed in the acceptance of the objects that are summed.

(P3) Nothing could be considered more ontologically innocent than the request to accept something identical to things already accepted.

(C) Mereology is ontologically innocent.

(StrongCom) The predicate "are" used for the composition relation is literally the plural for of the "is" of identity.
∀X ∃y (y = the sum of the X) → y = X

(StrongCom') The predicate "are" used for the composition relation is analogous to the plural form of the "is" of identity.
∀X ∃y (y is the sum of the X) → y = X

(WeakCom) The predicate "are" used for the composition relation is analogous to the plural form of the "is" of identity.
∀X ∃y (y is the sum of the X) → y = X

3. Some arguments pro and cons (StrongCom), specifically Lewis, and Van Inwagen arguments against (StrongCom) (we argue for (T1) and (T2)).

- Lewis I against (StrongCom):

(WeakCom) The predicate "are" used for the composition relation is analogous to the plural form of the "is" of identity.
∀X ∃y (y is the sum of the X) → y = X

- A reply: X = y in (ISP) has a distributive reading, i.e. each of the X is identical to y, whereas when y is a sum of the X identity has a collective reading.

- A reply to the reply: collective identity is not a genuine many-one relation.

- Lewis II against (StrongCom):

(IdIn) ∀x ∀y (x = y ↔ ∀F (F(x) ↔ F(y)))

"Even though the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernability of identicals... What is true of the many is not exactly what is true of the one. After all they are many while it is one" [Lewis 1991: 87].

- A reply: A justification of (StrongCom) is to argue that to strictly count the many is to loosely count the one.

- Suppose a man owned some land which he divides into six parcels. Overcome with enthusiasm for [the denial of composition as identity] he might try to perpetrate the following scam. He sells off the six parcels while retaining ownership of the whole. That way he gets some cash while hanging on to his land. Suppose the six buyers of the parcels argue that they jointly own the whole and the original owner now owns nothing. Their argument seems right. But it suggests that the whole was not a seventh thing" [Baxter 1988: 579].

- Lewis against (StrongCom)

(1) There is a cat, Mina, which is sleeping.
(2) There is a mouse, Gino, which is dancing.
(3) There is a sum of the mouse Gino and the cat Mina, Gina.

(P1) Composition – a many-one relation – is like identity.
(P2) The commitment to sums is already presupposed in the acceptance of the objects that are summed.

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"Given a prior commitment to cats, say, a commitment to cat-sums is not a further commitment. The sum is nothing over and above the cats that compose it. It is just them. They just are. Take them together or take them separately, the cats are the same portion of Reality either way. Commit yourself to their existence all together or at one time, it is the same commitment either way... I say that composition... is like identity. The 'are' of composition is, so to speak the plural form of the 'is' of identity. Call this the Thesis of Composition as Identity. It is in virtue of this thesis that mereology is ontologically innocent: it commits us only to things that are identical, so to speak, to what we were committed to before" [Lewis 1981: 81-82].

(P1) Composition – a many-one relation – is like identity.
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4. A reply to the reply: wrong examples

- A reply: there are "hybrid" uses of is/are in terms of "... is one of..." or in some other similar way such as the definition of identity in terms of overlapping?

- Lewis pro (WeakCom): The predicate "are" used for the composition relation is analogous to the plural form of the "is" of identity.

- Lewis anti (WeakCom): The predicate "are" used for the composition relation is not analogous to the plural form of the "is" of identity.

- Lewis pro (StrongCom): Just as if you fully describe the X you fully describe their sum.

- Lewis anti (StrongCom): Just as if you fully describe something identical to x, likewise you fully describe something identical to y.

Van Inwagen: how should we define the "hybrid" form "is/are" in terms of "... is one of..." or in some other similar way such as the definition of identity in terms of overlapping?

- There is something the mouse Gino is

- There is something the cat Mina is

- There is a mouse, Gino, which is dancing.

- There is a mouse, Gino, which has the capacity of two cups.

- There are some Germans.

- It (the sum) is just them (the cats composing it).

- They (the cats composing it) just are it (the sum).

- The sum y of the X is just the X,

- The X are just the sum y of the X.

- Tully is Cicero.

- x is y.

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The conclusive objection to Lewis’s argument of the similarity is that it is just a petitio principii.

Conclusion: arguments for the ontological innocence of mereology – both those based on (StrongCom) and those based on (WeakCom) – are not conclusive (T1). Moreover, we have argued that some arguments against the ontological innocence of mereology show a certain ambiguity in the innocence thesis itself. Some defenses of the innocence seem to implicitly presuppose that the sum of certain objects X is not a genuine entity (T2).

5. Let T be a theory of sets of individuals. The language L of T is a first order language with identity and with two kinds of variables:

- x, y, z, ... variables for individuals;
- α, β, γ, ... variables for sets;
- ∈ is the membership symbol.

Atomic formulas have the following form:

\[ \exists x (x = a) \]

\[ \exists x (x = b) \]

\[ \exists x (x = a \land x = b) \]

Complex formulas are defined in the usual way. Axioms of T are:

- Extensionality (ES) \[ \alpha = \beta \iff \forall x (x \in \alpha \iff x \in \beta) \]
- Comprehension (Com) \[ \exists x \forall y (y \in x \iff A(y)) \]

where A(x) is any propositional function of L. It is possible to give a mereological interpretation of T.

Observe that T is a substantive fragment of set theory, i.e. the one that grounds the principal model semantics of second order logic. Because of such ground Quine notoriously argues that second order logic is a wolf in sheep’s clothing. That means that second order logic is set theory in logic’s clothing. Since it is possible to give a mereological interpretation of T, Lewis’ assimilation of mereology to logic seems to be subject to the same objections (our T4 thesis).

6. What about the ontological innocence of mereology? Lewis’ ambiguity in the use of the term “sum”.

On one side, Lewis seems to argue that, given certain objects X, referring to their sum is ontologically innocent because there is not a new entity as referent of the expression “the sum of the X”. This seems to be the only way to make intelligible, and plausible, the statement that:

\[ \exists x \forall y (y \in x \iff A(y)) \]

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